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ADP013975

TITLE: Radiation by Self-Oscillating Relativistic Charged Particle Moving Along Periodic Structure

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TITLE: 2002 International Conference on Mathematical Methods in Electromagnetic Theory [MMET 02]. Volume 2

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RADIATION BY SELF-OSCILLATING RELATIVISTIC CHARGED PARTICLE MOVING ALONG PERIODIC STRUCTURE

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ABSTRACT

The new elementary radiation mechanism due to the oscillatory character of a radiation friction force appearing when a relativistic charged particle moves along a periodic structure without external fields is studied by analytical methods. The equation of motion for the charged particle driven by the radiation friction force is solved by the perturbation method. It is shown that the non-synchronous spatial harmonics of the Cherenkov-type radiation (CR) can cause an oscillatory motion of the particle, which therefore generates an undulator-type radiation (UR). In the frequency range where the diffraction of the generated waves is essential, the radiation manifests itself in the interference of CR and UR. The undulator radiation takes place only in that spectral region where the wave diffraction can be neglected.

As known, a charged particle moving with a constant velocity along a periodic structure emits Cherenkov-type radiation [1]. The fields of this radiation called wake-fields can be expressed as spatial-harmonic series according to the Floquet theorem. The action of the synchronous with the particle spatial harmonics of the self-wakefield on the particle results in energy losses associated with CR. The non-synchronous spatial harmonics can cause the oscillatory particle motion resulting in generating the undulator-type radiation. This radiation is subject for discussing in this article.

As a periodic structure, we will consider a hollow corrugated waveguide with metallic surface. Suppose that a particle of charge e and of mass m moves with an ultrarelativistic velocity $\mathbf{v}$ along the structure with the period D. The radiation friction force and the radiation power are sought for.

By using the Hamilton's method developed in [2] we can obtain the radiation friction force as

$$\mathbf{F}(\mathbf{v}(t),\mathbf{r}(t),t)=-\frac{e^2}{4c^2V_{tot}}\sum_{\lambda}^{\omega_\lambda<c/r_0}\left\{\left[\mathbf{A}_\lambda(\mathbf{r}(t))-\frac{\mathbf{v}(t)\times rot\mathbf{A}_\lambda(\mathbf{r}(t))}{i\omega_\lambda}\right]e^{i\omega_\lambda t}\int_0^t\mathbf{v}(t')\mathbf{A}_\lambda^*(\mathbf{r}(t'))e^{-i\omega_\lambda t'}dt'+\right.$$
$$\left.+\left[\mathbf{A}_\lambda(\mathbf{r}(t))+\frac{\mathbf{v}(t)\times rot\mathbf{A}_\lambda(\mathbf{r}(t))}{i\omega_\lambda}\right]e^{-i\omega_\lambda t}\int_0^t\mathbf{v}(t')\mathbf{A}_\lambda^*(\mathbf{r}(t'))e^{i\omega_\lambda t'}dt'\right\}+\kappa.c. \quad (1)$$

where ω_λ is an eigenfrequency. Seeing the force of radiation friction does not depend on the particle size r_0, so $\omega_\lambda < c/r_0$ [2] (c is the velocity of light). $V_{tot}=MV_{cell}$, where we assume that the structure contains $M\to\infty$ cells of volume V_{cell} and is enclosed in a "periodicity box". $\mathbf{A}_\lambda(\mathbf{r})$ is the set of the eigenfunctions of the vector potential which can be represented in the Floquet form [1]

$$\mathbf{A}_\lambda(\mathbf{r}) = \sum_{n=-\infty}^{\infty} \boldsymbol{g}_\lambda^{(n)}(\mathbf{r}_\perp)e^{ih_n z}, \tag{2}$$

where $\boldsymbol{g}^{(n)}{}_\lambda(\mathbf{r}_\perp)$ is the amplitude of the n-th spatial harmonic; h is the parameter of the interval: $(-\pi/D \div \pi/D)$; $h_n = h + 2\pi n/D$ is the propagation constant. The set of (2) is limited above on frequency by the electron plasma frequency ω_{pe} in metal. As known, if $\omega_\lambda \sim \omega_{pe}$ the diffraction conditions in the periodic structure are violated. So, for the range $\omega_\lambda > \omega_{pe}$ the vector potential can be expanded in terms of the plane waves of free space

$$\mathbf{A}_{\lambda,l}(\mathbf{r}) = c\sqrt{4\pi}\boldsymbol{a}_{\lambda,l}e^{i\boldsymbol{k}_\lambda \mathbf{r}} \tag{3}$$

where $\boldsymbol{k}_\lambda$ is the wave propagation vector; $\boldsymbol{a}_{\lambda,l}$ is the real unit vector of polarization (l=1,2) perpendicular to $\boldsymbol{k}_\lambda$.

The equation of the motion driven by the force (1) can be solved approximately in ultrarelativistic limit. As a zeroth-order approximation, we will consider the motion with a constant velocity parallel to the stricture axis

$$\mathbf{v} = \mathbf{v}_0 = \mathrm{v}_0\mathbf{e}_z, \qquad \mathbf{r}(t) = \mathbf{r}_{0,\perp} + \mathbf{v}_0 t \tag{4}$$

In this case, inserting (4) and (2) into (1), we obtain the self-wake force of zeroth order in the frequency band $\omega_\lambda << \omega_{pe}$

$$\mathbf{F}(t) = -e^2 \sum_{p=-\infty}^{\infty} \boldsymbol{w}^{(p)}e^{ip\Omega t} + \kappa.c., \tag{5}$$

where we have introduced $\Omega \equiv 2\pi\mathrm{v}_0/D$ and defined the amplitude of the pth spatial harmonic of the wake function as

$$\boldsymbol{w}^{(p)} \equiv \frac{D\mathrm{v}_0}{4c^2 V_{cell}} \sum_{n=0}^{\infty}\sum_{\lambda'_j} \frac{g_{z,\lambda_j}^{(n)\,*}}{\left|\mathrm{v}_0 - \frac{d\omega_\lambda}{dh}\right|_{\lambda=\lambda_j}} \left[\boldsymbol{g}_{z,\lambda'_j}^{(n+p)} - i\frac{\mathrm{v}_0}{\omega_\lambda}\nabla_\perp g_{z,\lambda_j}^{(n+p)} - \frac{\Omega p}{\omega_\lambda}\boldsymbol{g}_{\perp,\lambda_j}^{(n+p)}\right] \tag{6}$$

Here $\boldsymbol{g}_\lambda^{(n)} \equiv \boldsymbol{g}_\lambda^{(n)}(\mathbf{r}_{0,\perp})$, and ω_{λ_j} satisfies the resonance conditions $\omega_\lambda - h\mathrm{v}_0 = n\Omega$.

The force (5) is a periodic function of time with the period D/v_0. The synchronous harmonic of this force, $-e^2 2w_z^{(0)}$, determines the energy losses associated with CR in the frequency band $\omega_\lambda << \omega_{pe}$.

In the region $\omega_\lambda > \omega_{pe}$ where the structure can be considered as free space, the radiation is absent in zeroth order approximation, i.e. at v_0=const.

If the particle is off-axis, it experiences a periodic action of the transverse component of the non-synchronous harmonics of the self-wake force ($\boldsymbol{w}_\perp^{(p)} \neq 0$). The radiation due to the periodic motion we will call undulator radiation. Solving the equation of the motion driven by the force (5) we find the corrected law of motion

$$\mathbf{v}(t) = \mathbf{v}_0 + \mathbf{v}_\perp(t) = \mathbf{v}_0 + ic\sum_{p\neq 0}^{\infty}\frac{{}^{(p)}}{p}e^{ip\Omega t}, \quad \mathbf{r}(t) = \mathbf{r}_{0,\perp} + \mathbf{v}_0 t + \delta\mathbf{r}_\perp(t) = \mathbf{r}_{0,\perp} + \mathbf{v}_0 t + \frac{c}{\Omega}\sum_{p\neq 0}^{\infty}\frac{{}^{(p)}}{p^2}e^{ip\Omega t} \tag{7}$$

where ${}^{(p)}$ is the dimensionless vector ${}^{(p)} \equiv \frac{2e^2}{mc\gamma\Omega}\left(\boldsymbol{w}_\perp{}^{(p)} + \boldsymbol{w}_\perp{}^{(-p)*}\right)$ and $|\alpha^{(p)}| << 1$.

Inserting Eq.(7) and Eq.(2) into Eq.(1) we can obtain the power radiation as

$$P \equiv -\lim_{t\to\infty}\frac{1}{t}\int_0^t \mathbf{v}(t')\mathbf{F}(\mathbf{v}(t'),\mathbf{r}(t'),t')dt' =$$

$$= \frac{e^2 D}{2V_{cell}}\sum_{n=0}^{+\infty}\sum_{\lambda_j}\frac{1}{\left|v_0 - \frac{d\omega_\lambda}{dh}\right|_{\lambda=\lambda_j}}\left[\left|\beta_0 g_{z,\lambda_j}^{(n)} + \frac{1}{2}\sum_{p\neq 0}\frac{\alpha^{(p)}}{p}\left(\frac{D}{2\pi p}\nabla_\perp g_{z,\lambda_j}^{(n+p)} - i g_{\lambda_j}^{(n+p)}\right)\right|^2 + O\left(\left|\alpha^{(p)}\right|^2\right)\right] \quad (8)$$

Here ω_{λ_j} is a frequency found from the equation $\omega_\lambda - h v_0 = n\Omega$, γ is Lorentz factor. (8) shows that in the region $\omega_\lambda << \omega_{pe}$, the radiation manifests itself in the interference between CR and UR.

For the region $\omega_\lambda > \omega_{pe}$ it is interesting to consider the radiation of a high energy charged particle satisfying the condition $\omega_{pe} << 2\Omega\gamma^2$. Inserting (7) and (3) into (1) and for simplicity applying the dipole limit $k_\lambda \delta \mathbf{r}_\perp(t) << 2\pi$ we obtain the UR power by analogy with (10)

$$P_U \equiv -\lim_{t\to\infty}\frac{1}{t}\int_0^t \mathbf{v}(t')\mathbf{F}(\mathbf{v}(t'),\mathbf{r}(t'),t')dt' = \frac{4e^6}{3m^2c^3}\gamma^2\sum_{p=1}^{p<<p_{\lim}}\left|\boldsymbol{w}_\perp^{(p)} + \boldsymbol{w}_\perp^{(-p)*}\right|^2 \quad (9)$$

where the number of harmonics in the sum is limited by the condition of the small value of the oscillation amplitude resulting in $p << p_{\lim} = 2\pi\gamma/\max\{\alpha^{(p)}\}$.

As follows from (9), the power grows up as square of the particle energy, so in the region $\omega >> \omega_{pe}$ the UR power can exceed the CR power emitted in the band $\omega << \omega_{pe}$. It should also be stated that, if a bunch of N electrons moves in the periodic structure and the bunch dimensions σ_z and $\sigma_\perp$ satisfy the conditions $\sigma_z << D/(2q\gamma^2)$ and $\sigma_\perp << D/(2q\gamma)$, then the radiation with the frequency $\omega < 2q\Omega\gamma^2$ is coherent. Moreover, for the range $\omega_{pe} << \omega < 2q\Omega\gamma^2$ the UR power would be proportional to N^4

$$P_U = \frac{4e^6 N^4}{3m^2c^3}\gamma^2\sum_{p=1}^{q}\left|\boldsymbol{w}_\perp^{(p)} + \boldsymbol{w}_\perp^{(-p)*}\right|^2 \quad (10)$$

Finally, it should be noted that in the future super-high-power electron rf linacs there will be beam energy loss associated with the undulator radiation emitted by the electrons in the fields of the spatial harmonics of both the accelerating mode [3] and the wakefield, due to the deviation of beams from the linac axis. On the other hand, the considered above radiation mechanism can be used in the undulators based on periodic RF waveguides without external fields, where the non-synchronous wake-harmonics of an electron bunch implies a wave pump. The development of such wake-field undulators with submillimeter periods may open new frontiers in generating X and gamma rays without employing external periodic magnetic fields and RF sources.

I am grateful to Ya.B. Fainberg for the proposed method of solution and fruitful discussions.

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